

Optimizing energy use in electric vehicles: A comparative study of optimization methods

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ABSTRACT

Electric and hybrid passenger vehicles offer the ability to recover braking energy and optimize power distribution, enabling significant improvements in efficiency compared to conventional vehicles. In this study, we quantify and compare the energy-saving potential of three optimization methods—mixed-integer linear programming (MILP), nonlinear programming (NLP), and dynamic programming (DP)—for a battery-electric vehicle following the Urban Dynamometer Driving Schedule (UDDS). A simplified longitudinal EV model with aerodynamic, rolling, and gravitational resistances is used to simulate vehicle dynamics, battery charging/discharging, and regenerative braking. The reference case enforces exact tracking of the UDDS speed profile, including late, hard braking at stops. Over a receding horizon of five seconds, MILP linearizes resistive forces and solves a linear program (≈ 16 ms per step), while NLP retains full nonlinear dynamics with a quadratic speed-deviation penalty (≈ 1.7 ms per step). DP serves as an offline global benchmark across a discretized state-of-charge and velocity grid. Results show that, relative to the baseline, NLP achieves the largest net-energy reduction (31.1 %), followed by DP (23.5 %) and MILP (18.2 %). NLP's ability to model exact aerodynamic drag and smoothly coast into stops yields the highest savings but requires more computational effort. MILP offers a compromise between efficiency and real-time feasibility on automotive ECUs, although its hard speed bounds induce oscillatory speed errors. DP, while globally optimal under exact speed tracking, is best suited as an offline reference. These findings suggest that hybrid approaches—such as DP-trained lookup tables or variable-efficiency NLP surrogates—could approach global optimality while meeting real-time constraints. Future work will validate these methods on

additional driving cycles and incorporate variable motor/regenerative efficiency maps to better align with real-world performance.

1. INTRODUCTION

Electric and hybrid passenger vehicles have spurred extensive research into strategies for minimizing energy consumption, motivated by environmental goals and the need to extend electric driving range (Gómez-Barroso, Vicente Makazaga, & Zulueta, 2025). Unlike conventional vehicles, EVs and HEVs offer new degrees of freedom such as regenerative braking and power split between engine and motor that can be optimized to improve efficiency. Researchers have explored various control and optimization approaches to reduce energy usage by adjusting driving speed, power distribution between power sources, regenerative braking intensity, and even gear selection in multi-gear transmissions. Recent studies range from global optimization methods that assume complete knowledge of the driving cycle, to real-time strategies embedded in vehicle controllers.

1.1 RECENT DEVELOPMENTS IN VEHICLE ENERGY OPTIMIZATION

In the past decade, energy management strategies for EVs and HEVs have rapidly advanced, moving beyond simple rule-based heuristics to more sophisticated optimization-based and learning-based techniques (Gómez-Barroso, Vicente Makazaga, & Zulueta, 2025). A broad trend is the pursuit of global optimality in energy use: many researchers formulate the problem as an optimal control or optimization task over a driving cycle or route. For example, dynamic programming (DP) has been used to compute the theoretical minimum energy or fuel consumption requirement for a predetermined trip. Such globally optimal solutions, while computationally intensive, serve as benchmarks to evaluate other strategies (Gómez-Barroso, Vicente Makazaga, & Zulueta, 2025). Other recent works employ mathematical programming formulations, including linear and nonlinear programming, to handle the HEV power split problem or EV speed profile optimization with high fidelity models (Braghin, Robuschi, Sager, Cheli, & Zeile, 2019) (Gómez-Barroso, Vicente Makazaga, & Zulueta, 2025). There is also growing interest in predictive and connected vehicle strategies: using lookahead information (from GPS or V2X communication) about road grade, traffic, and route to improve efficiency. Studies show that incorporating upcoming road slope data into

energy management can yield measurable fuel or energy savings. For instance, (Zhang, Pisu, Li, Tennant, & Vahidi, 2010) demonstrated that pre-viewing terrain elevations enables a hybrid vehicle to save additional fuel, with the benefit depending on speed and road profile. Overall, recent developments emphasize integration of vehicle data and advanced algorithms to push energy efficiency closer to its theoretical optimum under real driving conditions (Gómez-Barroso, Vicente Makazaga, & Zulueta, 2025). However, each approach comes with practical trade-offs, leading researchers to investigate a spectrum of methods.

1.2 SIGNIFICANCE OF BENCHMARKING MULTIPLE METHODS

Given the variety of optimization approaches, systematic benchmarking under consistent conditions is crucial. Direct comparisons of MIP, NLP and DP can illuminate trade-offs between optimality, computational cost, and robustness, helping practitioners select the most suitable methods. Benchmarking across varied driving scenarios; urban, highway and hilly would provide deeper insights into method-specific strengths and weaknesses (Gómez-Barroso, Vicente Makazaga, & Zulueta, 2025). Furthermore, combining methods within hybrid control architectures could unlock near-optimal real-time strategies by balancing global planning and fast local decision-making. A study that unifies and evaluates these approaches would address a major research gap and offer practical guidance for next-generation energy management systems.

1.3 OPEN CHALLENGES AND RESEARCH GAPS

Despite substantial progress, several open challenges remain in the quest for optimal vehicle energy management. Dynamic programming and fully non-linear MPC remain too slow for production ECUs; customised solvers must deliver sub-second runtimes in all conditions for real-time feasibility. Strategies tuned to specific cycles degrade when driver behaviour, weather, or component ageing diverge from assumptions; stochastic and RL-based schemes help but still lack reliable on-road calibration. Hybrids such as DP-trained neural policies or metaheuristic-tuned ECMS are promising yet largely unexplored. Furthermore, multi-objective considerations such as battery longevity, emissions, and thermal limits increasingly matter, but few energy-management systems handle such trade-offs online. Finally, most studies remain

simulation-only; systematic hardware-in-the-loop and public road campaigns are still scarce.

This study aims to quantify and compare the energy-saving potential of three optimization approaches - mixed-integer linear programming (MILP), non-linear programming (NLP) and dynamic programming (DP) - when applied to a standard passenger vehicle driven over the Urban Dynamometer Driving Schedule (UDDS).

2. CONTROL OPTIMIZATION METHODS

To minimize energy consumption, a variety of optimization techniques have been applied in prior studies. These include Mixed-Integer Programming, Nonlinear Programming, Dynamic Programming, and metaheuristic algorithms. In this section the focus is on how each method has been used and its suitability for real-time embedded implementation.

Mixed-Integer Programming (MIP) formulates discrete choices as integer variables and linearises the remaining drivetrain dynamics. The resulting Mixed Integer Linear Program (MILP) can be solved fast enough for short-horizon supervisory Model Predictive Control (MPC) on modern embedded CPUs (Gómez-Barroso, Vicente Makazaga, & Zulueta, 2025). According to that review, Yamanaka et al. (2022) demonstrated an online MILP power-split controller for connected HEVs, and Robuschi et al. (2020) reported fuel savings using iterative linear programming. The strengths of MIP in this context are exact treatment of discrete decisions and guaranteed convergence to the MILP optimum. Limitations include linearisation errors and rapidly increasing solve time as the time grid or integer count is refined (Gómez-Barroso, Vicente Makazaga, & Zulueta, 2025).

Nonlinear Programming (NLP) keeps the full non-linear problem, capturing aerodynamic drag, battery losses and engine maps without requiring linearisation. The well-known Pontryagin-based ECMS converts electric energy to an equivalent fuel cost and reaches $\approx 95\%$ of the DP optimum when its equivalence factor is adapted online (Zhu, Wu, & Xu, 2020). Direct transcription NLP is typically run offline; when real-time is required researchers pre-compute speed and torque trajectories or convexify the model for rapid MPC (Burtchen, Maurer, & Pickenhain, 2016). The strengths of NLP lie in its high model fidelity and transparent inclusion of complex constraints. It is limited by heavy computational load where full real-time NLP remains difficult on production ECUs (GómezBarroso, Vicente Makazaga, & Zulueta, 2025).

Dynamic Programming (DP) provides the global minimum energy for a fully known drive cycle and is therefore the gold-standard benchmark (Gómez-Barroso, Vicente Makazaga, & Zulueta, 2025). Its weaknesses are the curse of dimensionality and the need for full-cycle preview. Practical spin-offs include stochastic DP policies, DP-derived lookup tables and neural or reinforcement-learning surrogates trained on DP data that deliver millisecond decisions (Fan, o.a., 2024).

3. MATHEMATICAL FORMULATIONS

Section 3.1 describes our discrete-time longitudinal EV model, which captures aerodynamic, rolling and gravitational forces along with battery charge/discharge and regenerative braking. Sections 3.2–3.4 then present the optimisation formulations -receding-horizon MILP, direct-shooting NLP, and global-benchmark DP (with SoC and speed discretisation) used to compute energy-efficient speed trajectories.

3.1 DYNAMICS AND ENERGY MODEL

The energy model is based on the state and control variables presented in Table 1, together with the vehicles’ physical parameters given in Table 2. Values used in the simulation are also given in the tables.

Table 1: State and control variables.

| Symbol | Definition |
|--------------------------|--|
| v_k | Vehicle speed at time step k [m/s] |
| θ_k | Road slope angle at step k [rad] |
| SoC_k | Battery state of charge at step k [0-1] |
| | Regenerative braking efficiency [-] |
| | Motor efficiency 0.9 [-] |
| F_k | Applied traction force during $[k, k + 1]$ [N] |
| $F_{res}(v_k, \theta_k)$ | Total resistive force [N] |
| P_{wheel} | Mechanical power at the wheels [W] |

Table 2: Physical parameters.

| Symbol | Description | Value |
|----------------|--|----------------------------|
| m | Vehicle mass [kg] | 1610 kg |
| g | Gravitational acceleration [m/s ²] | 9.81 [m/s ²] |
| ρ | Air density [kg/m ³] | 1.225 [kg/m ³] |
| A | Frontal area [m ²] | 2.22 [m ²] |
| C _d | Aerodynamic drag coefficient | 0.23 |
| C _r | Rolling resistance coefficient | 0.012 |
| Q | Battery capacity [kWh] | 60 [kWh] |
| Δ | Regenerative deadband threshold | 300 [W] |

The driving force is modeled as the sum of the forces to overcome aerodynamic drag, rolling resistance and climbing resistance

$$F_{res}(v_k, \theta_k) = \frac{1}{2} \rho A C_d v_k^2 + mg C_r \cos(\theta_k) + mg \sin(\theta_k) \quad (1)$$

The battery power at step k is computed as

$$P_{bat,k} = \begin{cases} \frac{P_{wheel,k}}{\eta_{mot}} & P_{wheel,k} \geq 0 \\ 0 & P_{wheel,k} < 0, |P_{wheel,k}| \leq \Delta P_{db} \\ P_{wheel,k} \eta_{regen}(v_k) & P_{wheel,k} < -\Delta P_{db} \end{cases} \quad (2)$$

Where η_{regen} depends on speed. With discrete time steps $\Delta t = 1s$ the vehicle dynamics and battery State-of-Charge are modeled as:

$$v_{k+1} = v_k + \Delta t \frac{F_k - F_{res}(v_k)}{m_{eff}}, \quad SoC_{k+1} = SoC_k - \frac{P_{bat,k} \Delta t}{Q} \quad (3)$$

The rotational inertia of wheels and drivetrain is modeled as an equivalent mass applied only m_{eff} in the inertial term, ensuring correct accounting of acceleration/deceleration energies.

3.2 BASELINE PROBLEM

The reference solution is given by accurately following reference speed $v_k^* \Rightarrow F_k$ given by (2).

Net energy is

$$E_{ctrl} = \sum_{k=0}^{N-1} \max(P_{bat,k}, 0) \Delta t \quad (4)$$

3.3 MILP

For a rolling window $H = 5$ steps the resistance forces are linearised around v_{k+1}^* and the following Mixed-Integer Linear Program is solved:

$$\begin{aligned} \min_{F_{0:H-1}} \quad & \sum_{i=0}^{H-1} (\hat{\alpha}_i F_i + \hat{\beta}_i) \Delta t \\ \text{s.t.} \quad & v_{i+1} = v_i + \frac{\Delta t}{m_{eff}} (F_i - \hat{F}_{res,i}) \\ & |v_{i+1} - v_{k+i+1}^*| \leq 0.5 \\ & -F_{max} \leq F_i \leq F_{max} \end{aligned} \quad (5)$$

Where $\hat{\alpha}_i, \hat{\beta}_i$ are linearisation coefficients, integer variables enforce a three-way disjunction (motoring/deadband/regeneration), and regenerative efficiency η_{regen} is represented by an SOS2 piecewise-linear approximation.

3.4 NLP

By a single shooting method, the following non-linear program is solved explicitly:

$$\min_{F_0:H-1} \sum_{i=0}^{H-1} \left[\max(P_{bat,i}, 0) \frac{\Delta t}{3600} + w_v (v_{i+1} - v_{k+i+1}^*)^2 \right] \quad (6)$$

With box-bounds $|F_i| \leq F_{max}$. The weight parameter d_e is chosen to achieve an acceptable speed error compared to reference.

3.5 DP

The State of Charge is discretized in M grid points s_j . Transform from SoC index j to \hat{j} at step k is given by (1):

$$\hat{j} = \text{round} \left(j - \frac{P_{bat,k} \Delta t}{Q \Delta s} \right) \quad (7)$$

And at each timestep k we discretize the vehicle speed in L evenly spaced values with $\pm \Delta v$ around the reference speed. The cost is:

$$\ell_k(j, \hat{j}) = \max(P_{bat,k}, 0) \frac{\Delta t}{3600} + w_v (v_j - v_{ref}[k+1])^2 \quad (8)$$

Where $J_k(j)$ the minimal cumulative cost to reach SoC state j at time k forming a sequence. Bellman-recursion is then used to optimize this sequence:

$$J_{k+1}(j) = \min_i (J_k(j) + \ell_k(j, \hat{j})) \quad (9)$$

Backtracking gives global minimal positive battery energy $E_{DP} = J_n(j_N)$ as well as optimal $SoC^*(k)$.

4. RESULT

This section presents a comparison between the baseline (reference) scenario and the three optimization methods: MILP, NLP, and DP. Table 3 summarizes key energy metrics such as traction energy, recovered regenerative energy, net battery usage, and average energy consumption.

Table 4 shows the absolute and percentage-based energy savings compared to the reference. The NLP method demonstrates the greatest reduction in both net energy consumption and average consumption per kilometer, followed by MILP and DP.

Figure 1 provides a visual comparison of speed and battery state-of-charge (SoC) profiles over time.

Table 3: Energy and consumption results.

| | Reference | MILP | NLP | DP |
|-----------------------------|-----------|--------|--------|--------|
| Traction energy [Wh] | 2142.8 | 1713.5 | 1955.2 | 1799.0 |
| Regen recovered [Wh] | 362.3 | 254.2 | 730.7 | 439.8 |
| Net battery energy [Wh] | 1780.5 | 1459.3 | 1224.5 | 1359.3 |
| Average consumption [Wh/km] | 148.5 | 121.5 | 102.3 | 113.7 |

Table 4: Energy savings compared to reference

| | Δ Net energy [Wh] | Δ Consumption [Wh/km] | Savings [%] |
|------|--------------------------|------------------------------|-------------|
| MILP | -321.2 | -27.0 | 18.2 |
| NLP | -556.0 | -46.2 | 31.1 |
| DP | -421.2 | -34.8 | 23.5 |

5. DISCUSSION

NLP delivers the largest percentage savings because it retains the full nonlinear physics (aerodynamic drag, battery losses, motor efficiency maps) without linearisation. This allows it to exploit every opportunity for energy recuperation and efficient acceleration more fully than the MILP approach.

The residual velocity plot in figure (1) highlights a characteristic behaviour of the MILP method, where the optimal solution frequently oscillates within the acceptable speed deviation. This is a direct consequence of how the MILP problem is constructed where the deviation constraint $|v_k - v_k^*| \leq 0.5$ m/s is implemented as a hard bound. As long as the solution remains within this constraint all speeds are equally “free” from a cost perspective. The MILP solver exploits this flexibility, allowing the speeds to alternate between upper and lower bound to achieve a lower net energy. A proposed possible solution to this is implementing a cost in the objective function.

Although DP is globally optimal by design, in our implementation it is limited by the grid size and only small speed deviations allowed. The speed tracking penalty imposed on the cost function is a function of w_v , which was chosen to 4. This specific value was chosen arbitrarily to allow slight deviation from the reference speed. This parameter depends on the driving cycle and cannot be arbitrarily chosen to fit all cycles. The DP algorithm was also limited by a hard speed constraint of 1 m/s, this was done partly to speed up computation and to limit the solvers’ ability to stray too far away from the reference speed. This limits its real-world energy benefits compared to NLP and the receding-horizon MILP. In practice, relaxing the hard speed constraint or increasing the SoC and velocity grid size could recapture some of DP’s “lost” potential.

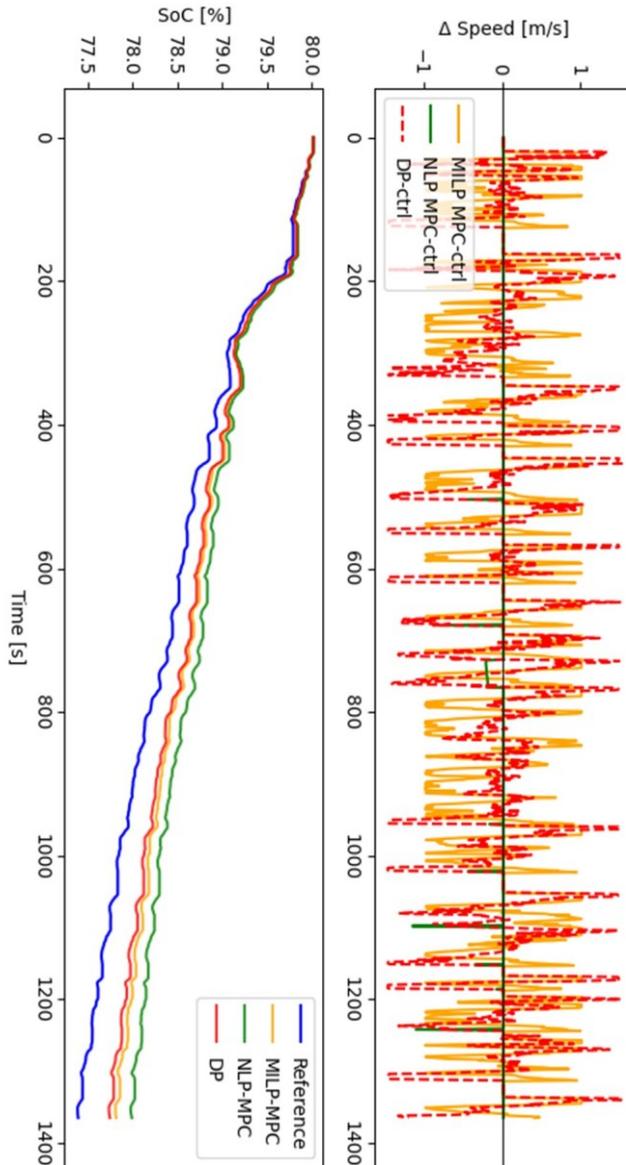


Figure 1: Time series comparing the reference case with each optimization method

Nonlinear programming (NLP) delivered the largest percentage savings (31.1 %) by retaining the full nonlinear physics (aerodynamic drag, battery losses, etc.) and yet each receding-horizon solve took only around 1.7 ms. Mixed-

integer linear programming (MILP) achieved 18.2 % savings but required roughly 16 ms, per 1 s window placing it on the upper edge of a typical automotive ECU budget (10-20 ms for a 1 Hz loop). Dynamic programming (DP), at 2788 seconds total, remains strictly an offline benchmark in line with the NLP and MILP results.

The simpler MILP scheme requires less maintenance and validation, whereas full NLP demands high-fidelity models and very stable solvers, which can be harder to certify in automotive applications where parameters can change quickly.

The Battery model is limited by the assumption that the parameters are constant, in reality these vary with battery age and temperature. If this is proven to be a substantial source of error the NLP method could be built upon with a non-linear function of, while MILP would require further linearisation of these parameters. η_{regen} was expanded into a small efficiency map dependent on vehicle speed which improved performance but was limited in scale.

Other studies such as (Hamednia, Sharma, Murgovski, & Fredriksson, 2022) have reported savings of 15.7% for a mixed route when an SQP-NLP based method is applied on an electric vehicle. Efficiencies of regenerative braking and engine were modeled using nonlinear functions of motor speed and torque. The difference in savings is likely due to the reference simulation exactly following the UDDS speed profile exactly. This results in braking as late and as hard as possible at every stop, which maximizes regenerative torque. In contrast the optimized strategies tend to coast more, regenerating less peak energy. As a result, the percentage reduction of the optimized methods artificially inflate.

Results are tied to the UDDS drive cycle. Savings might shift in stop-and-go city traffic, highway cruising, or hilly terrain. It is therefore recommended that the models are validated and compared on different cycles.

Future work should include validation of these methods on stop-and-go urban cycles such as the WLTC city segment, High-speed freeway cycles such as LA-92 and hilly terrain where grade preview is critical. Incorporation of variable efficiency maps for η_{regen}, η_{mot} could better position savings results against published literature. Finally exploring hybrid strategies such as DP-trained lookup tables and machine-learning surrogates could reveal whether a mixed-method approach can capture most of the offline optimality while still running in real time.

6. CONCLUSION

This study set out to compare three optimisation approaches: MILP, NLP, and DP on the UDDS cycle. We found that NLP achieved the greatest net energy savings (31.1 %), MILP delivered near-real-time performance with 18.2 % savings, and DP at 23.5 %.

For applications demanding strict real-time operation, MILP-based MPC is already feasible on modern ECUs. However, systems seeking maximum efficiency should consider hybrid or full NLP strategies as computational power grows.

Future work should integrate predictive data (e.g. V2X information) and broader set of driving conditions. As well as developing hybrid control schemes that combine each method's strengths which could yield both real-time capability and near-optimal efficiency. Balancing optimization accuracy against real-time feasibility is critical for the sustainable deployment of future electric and hybrid vehicles.

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